**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Z score = X-myu/Sigma

X = 60min

myu = 55min

Sigma = 8 min

Z = 60-55/8 = 0.625

Cummulative Probability Q (Normal Distribution) of 0.625 = 0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Total = 400

Mean myu = 38

SD = sigma = 6

Z score of 44 = 44-38/6 = 1

From left Z table = 1 = 0.8413 = 84.13%

84.13% in 400 = 63 people.

Z score of 38 = 38-38/6 = 0

From left Z table = 0.5000 = 50%

50% in 400 = 200 people.

People age between 38 to 44 = 200 – 63 = 137

Hence statement A is false.

Z score of 30 = 30-38/6 = -1.33 = 0.09175 =9.175% = 36 out of 400

Hence statement B is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *independent* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS:

X = N(μ1, σ12), and Y ∼N(μ2, σ22 )

X + Y = N(μ1 + μ2, σ12 + σ22);

X − Y =N(μ1 − μ2, σ12 + σ2^ )

Similarly if Z = aX + bY , where X and Y are as mentioned above

then Z = N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Hence ;

2X 1= N(2 u,4 σ2)

X1+X2 = N(μ + μ, σ2 + σ2 ) = N(2 myu, 2σ2)

If combined;

=2X1-(X1+X2) = N( 4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans.

The Probability of value a and b should be 0.99

So the Probability of going wrong area is 1-0.99 = 0.01

The Probability towards left from a = -0.005 ( 0.01/2)

The Probability towards right from b = +0.005 ( 0.01/2)

calculate X, the random variable at a and b

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

= Z=(X-μ) / σ

From the Z table

For 0.005 the Z Value is -2.57

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.5

Option D.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The profit given as two separate divisions but the profit is required for the total company hence add the profits of both divisions.

P= N(5+7,32+42)

P= N(12,52)

95% lies between 1.96 SD of the mean

=(12-1.96\*5,12+1.96\*5)

=($2.2M,$22.8M)

=(Rs.99M,Rs.1026M)

B) Fifth percentile

P(Z<=P-12/5) = 0.05

P-12/5 = -1.644

P = 12-8.22 = 3.78

=$3.78M

=Rs.170.1M

C)According to the above profits

Division A has probability of making more loss.